MAT123 MATHEMATICS I COURSE INFORMATION

- **Instructor:** Prof. Dr. Bülent Saraç

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- **Email:** bsarac@hacettepe.edu.tr

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- **Email:** bsarac@hacettepe.edu.tr
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- **Office Hours:** Wednesday 12:40-13:40 or by appointment

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- **Website:** https://bulentsarac.com

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- **Presentations:** delivered at classes are uploaded weekly.

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- **Recommended Textbooks:**
  - Calculus: A Complete Course, 9th Edition by R. A. Adams and C. H. Essex, Pearson Education.
  - Thomas' Calculus, 14th Edition by G. B. Thomas, M. D. Weir and J. H. Hass, Pearson Education.

MAT123 MATHEMATICS I COURSE INFORMATION

Syllabus:

- Preliminaries
- Limits and Continuity
- Differentiation
- Applications of Differentiation
- Integration
- Techniques of Integration
- Applications of Integration
- Sequences and Series

MAT123 MATHEMATICS I

Lecture 01: Preliminaries

Outline

Real Numbers and the Real Line

The Absolute Value

Cartesian Coordinates in the Plane

Graphs

Straight Lines

Graphs of Quadratic Equations

Real Numbers and the Real Line

Real Numbers and the Real Line

Real numbers are numbers that can be expressed as decimals.

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$$\begin{aligned} 5 &= 5.00000\ldots \\ -\frac{3}{4} &= -0.75000\ldots \\ \frac{1}{3} &= 0.33333\ldots \\ \sqrt{2} &= 1.41421\ldots \\ \pi &= 3.14159\ldots \end{aligned}$$

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Real Numbers and the Real Line

Real numbers can be represented on a number line, where each point corresponds to a unique real number.

Real Numbers and the Real Line

- ALGEBRAIC PROPERTIES
- ORDER PROPERTIES
- THE COMPLETENESS PROPERTY

Real Numbers and the Real Line

Order in Reals

$a < b$ or $b > a$

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Real Numbers and the Real Line

Order Properties

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Remark

The rules 1–4 and 6 (for $a > 0$) also hold if $<$ and $>$ are replaced by $\leq$ and $\geq$.

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(iv) the **irrational numbers**, namely, those real numbers that cannot be expressed as a fraction of two integers, such as $\sqrt{2}$ and π.

Example

Show that each of the numbers **(a)** $1.323232\ldots = 1.\overline{32}$ and **(b)** $0.3405405405\ldots = 0.3\overline{405}$ can be expressed as a quotient of two integers.

An **interval** is a set of real numbers that contains all numbers between any two numbers in the set. Intervals can be classified as follows:

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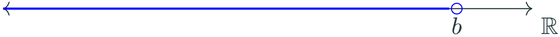

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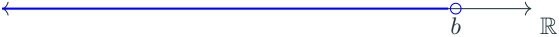

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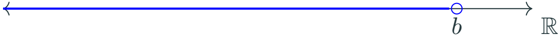

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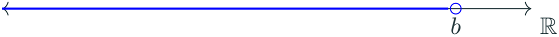

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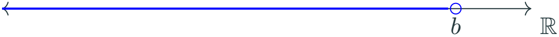

  - (a, ∞) includes all numbers greater than a.

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  - $(-\infty, a]$ includes all numbers less than or equal to a.

  - $[b, \infty)$ includes all numbers greater than or equal to b.

Example

Solve the following inequalities. Express the solution set in terms of intervals and graph them:

(a) $2x - 1 > x + 3$ (b) $-\frac{x}{2} \geq 2x - 1$ (c) $\frac{2}{x-1} \geq 5$

Example

Solve the systems of inequalities and express the solution set in terms of intervals:

(a) $3 \leq 2x + 1 \leq 5$ (b) $3x - 1 < 5x + 3 \leq 2x + 15$

Example (Quadratic Inequalities)

Solve the inequalities:

(a) $x^2 - 5x + 6 < 0$ (b) $2x^2 + 1 > 4x$

Example

Solve the inequality

$$\frac{3}{x-1} < -\frac{2}{x}$$

and graph the solution set.

The Absolute Value

The Absolute Value

The **absolute value** of a real number x, denoted by $|x|$, is defined as follows:

$$|x| = \begin{cases} x, & \text{if } x \geq 0, \\ -x, & \text{if } x < 0. \end{cases}$$

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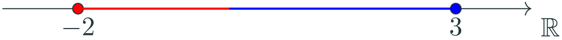
- $| x | \geq 0$ for all $x \in \mathbb{R}$.
- $| x | = 0$ if and only if $x = 0$.

The Absolute Value

Distance Interpretation

- The absolute value of a real number x can be interpreted as the distance from x to 0 on the real line.



$|-2| = 2$ $\quad$ $|3| = 3$

The Absolute Value

Distance Interpretation

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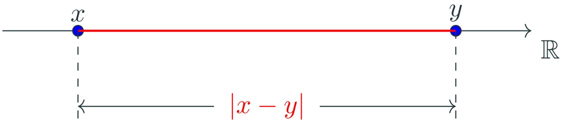
- $|\, x - y \,|$ represents the distance between the points x and y on the real line.



- $|-a| = |a|$ for all $a \in \mathbb{R}$.
- $|ab| = |a||b|$ for all $a, b \in \mathbb{R}$.
- $|a/b| = |a|/|b|$ for all $a, b \in \mathbb{R}$ with $b \neq 0$.
- $|a \pm b| \leq |a| + |b|$ for all $a, b \in \mathbb{R}$ (Triangle Inequality).

To solve an equation or inequality involving absolute values, we use the following properties:

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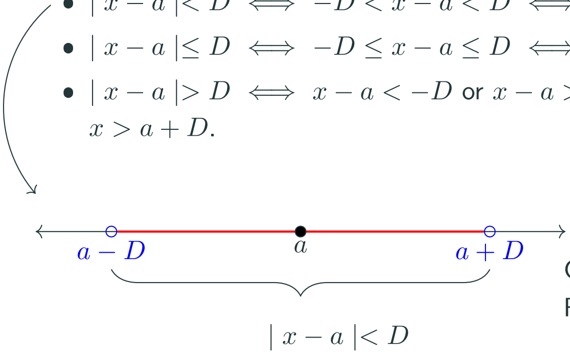
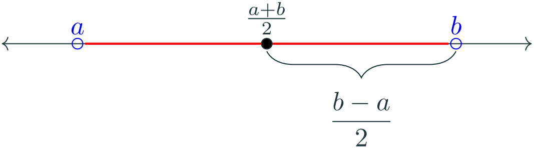


CENTER $= a$

RADIUS $= D$

The Absolute Value



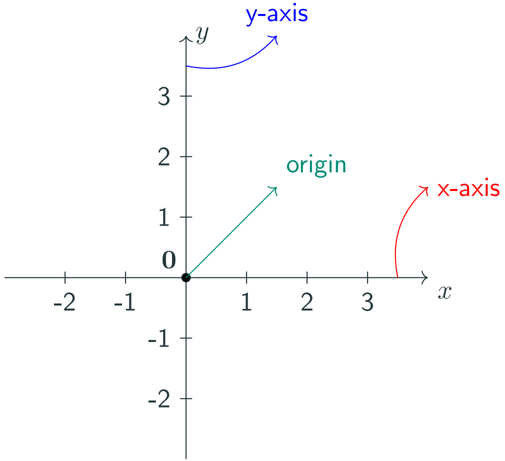
$$\left|x - \frac{a+b}{2}\right| < \frac{b-a}{2}$$

Example

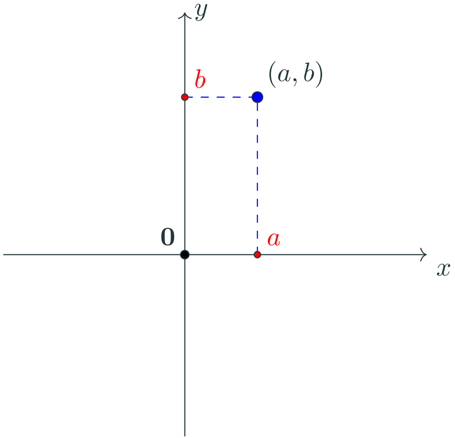
What values of x satisfy the inequality $\mid 5 - \dfrac{2}{x} \mid < 3$?

Cartesian Coordinates in the Plane

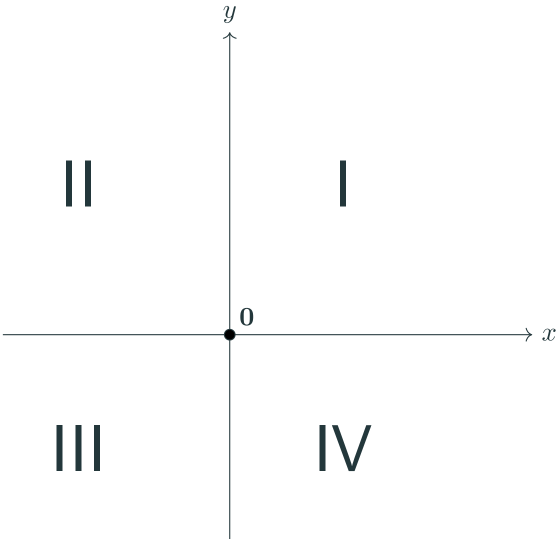
The Cartesian plane is a two-dimensional space defined by two perpendicular axes: the horizontal axis (x -axis) and the vertical axis (y-axis).



We locate a point in the plane by its **Cartesian coordinates** (a, b), where a is the x-coordinate and b is the y-coordinate. The point (a, b) is represented as follows:

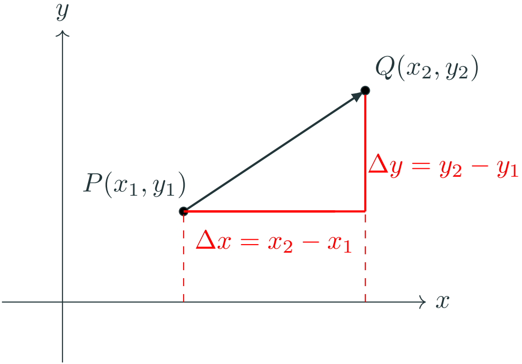


The coordinate axes divide the plane into four quadrants:



Cartesian Coordinates in the Plane

Increments and Distances



The distance from P to Q is given by the formula:

$$d(P,Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Graphs

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The **graph** of an equation (or inequality) in two variables x and y is the set of all points in the Cartesian plane that satisfy the equation (or inequality).

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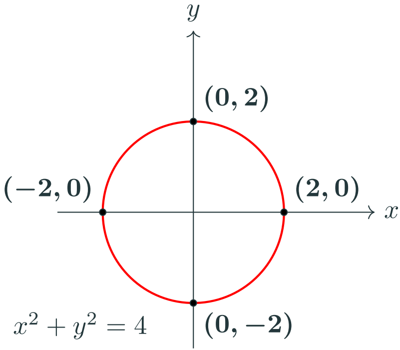

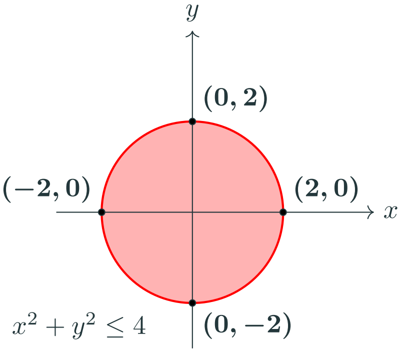

Straight Lines

Straight Lines

Definition.

A **straight line** is the graph of a linear equation in two variables, which can be written in the form

$$Ax + By = C$$

where A, B, and C are constants, and A and B are not both zero.

Straight Lines

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on a line with equation $Ax + By = C$. Then:

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Straight Lines

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on a line with equation $Ax + By = C$. Then:

$$\begin{array}{rr} & Ax_2 + By_2 = C \\ - & Ax_1 + By_1 = C \\ \hline & A \cdot \Delta x + B \cdot \Delta y = 0 \end{array}$$

Straight Lines

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on a line with equation $Ax + By = C$. Then:

$$
\begin{array}{rl}
 & Ax_2 + By_2 = C \\
- & Ax_1 + By_1 = C \\
\hline
 & A \cdot \Delta x + B \cdot \Delta y = 0
\end{array}
$$

$$\Big\downarrow\, B \neq 0$$

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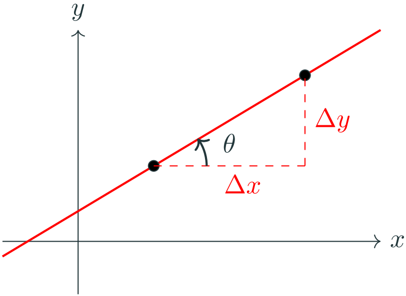
Slope of the line $= m = -\frac{A}{B}$

Straight Lines

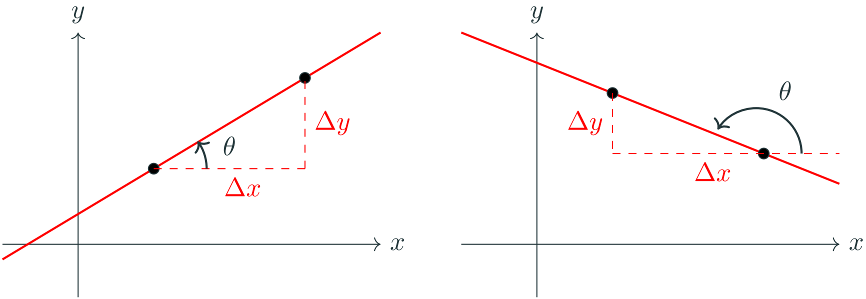
Inclination of a Line



The slope of the line is given by the formula:

$$m = \frac{\Delta y}{\Delta x}$$

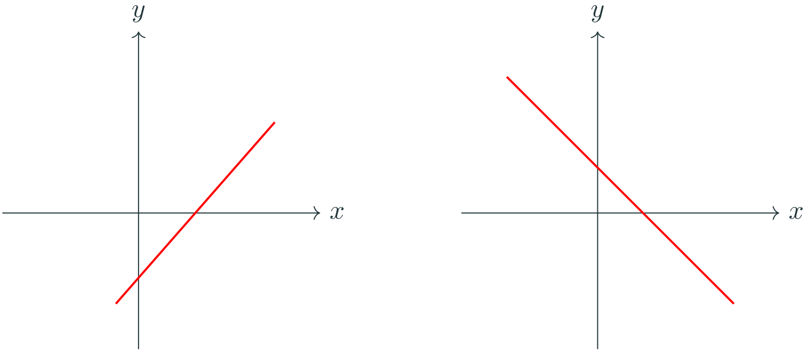
The **inclination** of a line is the angle θ that the line makes with the positive x-axis.



The slope of the line is given by the formula:

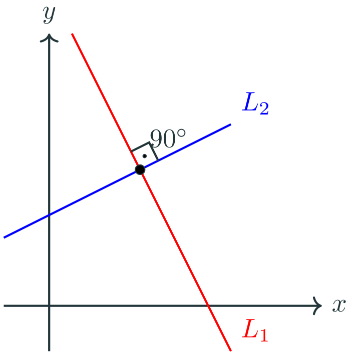
$$m = \tan(\theta) = \frac{\Delta y}{\Delta x}$$

Straight Lines

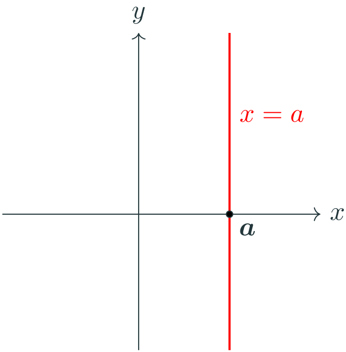


slope > 0 slope < 0

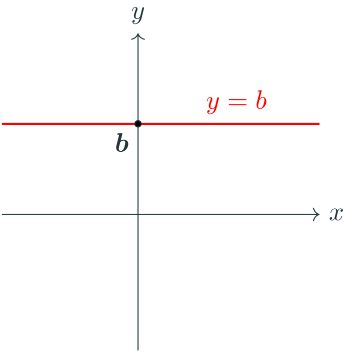
Two lines are said to be **perpendicular** if the angle between them is $90°$.



$$m_1 \cdot m_2 = -1$$



y
$x = a$
x
$\boldsymbol{a}$

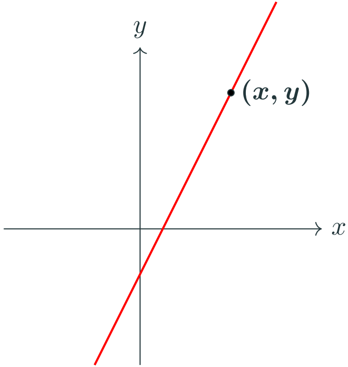


y
$y = b$
$\boldsymbol{b}$
x

The equation of a line in **standard** form is given by:

$$Ax + By = C$$

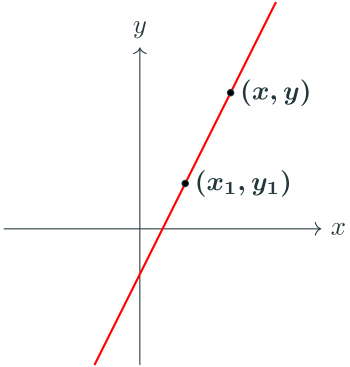
where A, B, and C are constants.



The equation of a line in **point-slope** form is given by:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is a point on the line and m is the slope.

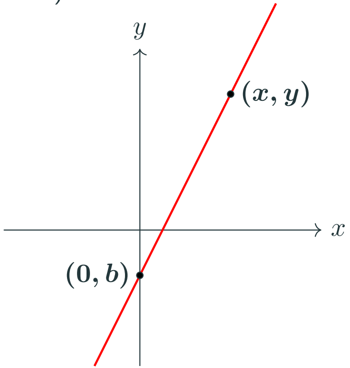


Straight Lines
Equations of Lines

The equation of a line in **slope-intercept** form is given by:

$$y = mx + b$$

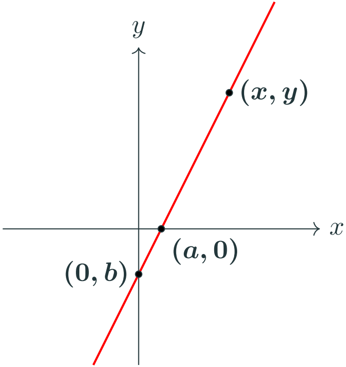
where m is the slope of the line and b is the y-intercept (the point where the line crosses the y-axis).



The equation of a line in **intercept** form is given by:

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a is the x-intercept and b is the y-intercept.



Example

Find the equation of the line that passes through the points $(1, 2)$ and $(3, 4)$.

Solution

Example

Find the equation of the line that passes through the points $(1, 2)$ and $(3, 4)$.

Solution

The slope of the line can be calculated using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, 4)$.

Example

Find the equation of the line that passes through the points $(1, 2)$ and $(3, 4)$.

Solution

The slope m is given by:

$$m = \frac{4-2}{3-1} = \frac{2}{2} = 1$$

Example

Find the equation of the line that passes through the points $(1, 2)$ and $(3, 4)$.

Solution

Now that we have the slope, we can use the point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

Substituting $m = 1$, $x_1 = 1$, and $y_1 = 2$, we get:

Example

Find the equation of the line that passes through the points $(1, 2)$ and $(3, 4)$.

Solution

$$y - 2 = 1(x - 1)$$

Simplifying this equation, we find:

$$y - 2 = x - 1 \implies y = x + 1$$

Example

Find the equation of the line that passes through the points $(1, 2)$ and $(3, 4)$.

Solution

Thus, the equation of the line is:

$$y = x + 1$$

Graphs of Quadratic Equations

A **circle** is the set of all points in the plane that are at a fixed distance (the radius) from a given point (the center). The equation of a circle with center (h, k) and radius r is given by:

$$(x - h)^2 + (y - k)^2 = r^2 \qquad (\star)$$

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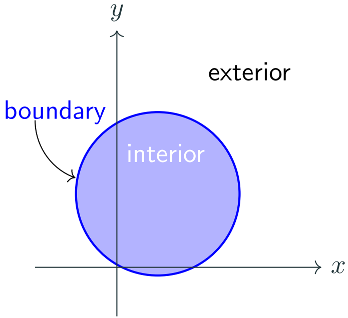
Expanding the equation $(\star)$, we get:

$$x^2 + y^2 - 2hx - 2ky = r^2 - h^2 - k^2$$



A **disk** is the set of all points in the plane that are at a distance less than or equal to a fixed distance (the radius) from a given point (the center). The equation of a disk with center (h, k) and radius r is given by:

$$(x-h)^2 + (y-k)^2 \leq r^2$$

Graphs of Quadratic Equations

Parabolas

A **parabola** (with principal axis parallel to the y-axis) is the graph of a quadratic equation of the form

$$y = ax^2 + bx + c$$

where a, b, and c are constants and $a \neq 0$.

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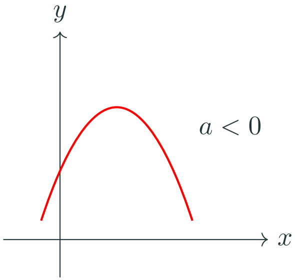
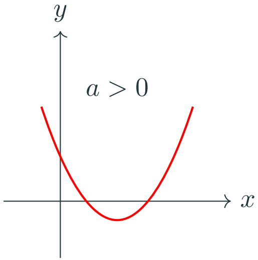
where a, b, and c are constants and $a \neq 0$.The graph of a parabola is U-shaped and can open either upward or downward depending on the sign of the coefficient a.





Graphs of Quadratic Equations

Parabolas

The vertex of a parabola is the point where the curve changes direction. The x-coordinate of the vertex can be found using the formula:

$$x_v = -\frac{b}{2a}$$

The corresponding y-coordinate can be found by substituting x_v back into the original equation.

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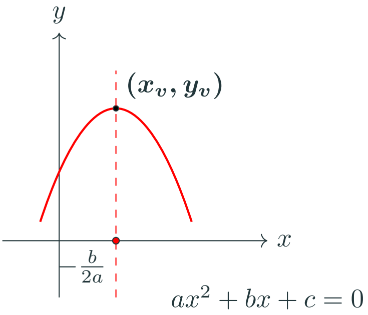

The axis of symmetry of the parabola is the vertical line that passes through the vertex, given by the equation:

$$x = -\frac{b}{2a}$$

The equation $ax^2 + bx + c = 0$ can be transformed into the vertex form by completing the square:

$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

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The graph of a parabola can be transformed by changing the coefficients a, b, and c in the equation $y = ax^2 + bx + c$.

- Changing a affects the width and direction of the parabola:
  - If $|a| > 1$, the parabola is narrower.
  - If $0 < |a| < 1$, the parabola is wider.
  - If $a < 0$, the parabola opens downward; if $a > 0$, it opens upward.
- Changing b affects the position of the vertex along the x-axis.
- Changing c shifts the parabola vertically.

The graph of an equation can be shifted horizontally or vertically by interchanging the x and y variables with $x - c$ and $y - d$, respectively. The effect of this transformation is as summarized in the table below, where c and d are **positive constants**:

Transformation	New Equation	Effect
Horizontal shift	$x \leftrightarrow x - c$	Moves the graph right by c units
Horizontal shift	$x \leftrightarrow x + c$	Moves the graph left by c units
Vertical shift	$y \leftrightarrow y + d$	Moves the graph down by d units
Vertical shift	$y \leftrightarrow y - d$	Moves the graph up by d units

Example

Describe the graph of the equation $y = x^2 - 4x + 3$.